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WORKING PAPER SERIES

WITHDRAWAL CAPACITY IN THE FACE OF EXPECTED AND UNEXPECTED HEALTH AND AGED-CARE EXPENSES DURING RETIREMENT

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CLUSTER PROJECT 7: MODELLING RETIREMENT OUTCOMES FOR ALL AUSTRALIANS

Abstract

We examine the impact of accounting for costs associated with age-related health treatment and aged-care services during the retirement phase on retirement income levels, income stability and longevity risk. To measure the impact of such costs on income sustainability and longevity, we simulate asset return data using historical bootstrap simulation to derive an optimal withdrawal income during retirement using dynamic optimisation techniques. We show that the greatest risk to income sustainability occurs when unexpected health costs translate into greater longevity, particularly for conservative investors. Paradoxically, this means that high costs associated with health treatment may result in a longer life however without a commensurate adjustment in asset allocation towards assets with a greater risk-return profile; it also risks premature wealth depletion. We further show that the optimal withdrawal rate is highly sensitive to the timing of health costs and moderately sensitive to later-life aged care costs. In response to this risk, we find that for a broad set of circumstances, the risk of premature ruin can be mitigated through a dynamic lifecycle strategy during the retirement phase.

Key words: Defined-contribution pension plan; Goal-oriented investing; Asset allocation; Dynamic lifecycle investing.

JEL: G11;

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1.0 Introduction

Fundamentally the economic wellbeing of individuals is largely determined by their command over economic resources including the income and wealth that is available to support their consumption of goods and services. This includes the funding of retirement. Individual retirement planning is a critical practice observed in most modern societies as the fiscal burden on the State to fund retirees is increasingly placed under pressure. Saving, wealth management and prudent planning has allowed individuals to increasingly finance at least part of their retirement, with a decreasing reliance on the State. Many older people also own their homes and have accumulated other assets which can be used in retirement to support living standards. It is natural for the cost burden of age-related health treatment and aged-care services to also eventually shift to individuals. However individual wealth planning and the impact on income sustainability and stability in reaction to this shift in liability has escaped systematic analysis.

It is well documented that the profile of the population is ageing within most developed countries. Ageing naturally affects individuals across a number of domains including physical and mental health, housing, income security, opportunities for social and economic participation, including labour force participation, and wealth management priorities. The decline in defined benefit (DB) pension plans and the transition to defined contribution (DC) plans is gradually expanding liability management from specifically funding retirement income to the financing of a wider range of social responsibilities including age-related health treatment, appropriate housing and aged-care management services and facilities.

Typical individual lifecycles comprise a period of employment followed by a period of retirement. Increasingly it falls to individuals to reallocate consumption from their working life to retirement if they wish to enjoy a financial security and avoid poverty in old age. DC pension plans can achieve this reallocation in a way that is consistent with the preferences of the individual plan member.

To achieve this there are generally three key preferences individuals take into account. First, preference is favoured towards an ability to smooth consumption across different possible states of nature within any given time period (asset diversification). Second, preference appears to be simultaneously awarded to the ability to smooth consumption across different time periods (temporal diversification). Third, the tension between current versus future consumption necessarily means that saving for retirement and other costs involves the sacrifice of certain consumption today in exchange for uncertain consumption in the future. In the literature the main elements of uncertainty remain manifest in both future labour income and the returns on the assets in which the retirement savings are invested. However future liabilities for both age-related health treatment and aged-care facilities are seldom identified by retirees as forming an essential cash flow need later in life (Quine and Carter, 2006). Indeed it is arguable that investors don’t tend to think of retirement consumption as a liability at all. Investors may count their superannuation portfolio as an asset but often forget to count the liabilities for which the asset is held.

DC plan workers intuitively (rather than systematically) form a view on both the trade-off between consumption in different states of nature in the same time period and the trade-off between consumption and consumption variability in different time periods. Of course, attitude and expectations related to these trade-offs will influence the optimal funding and investment strategies of the pension plan. Explicit consideration of potential costs incurred towards the end of one’s life tends to be an unsavoury reality and so remains largely ignored by financial advisors and retirees until the immediate need arises to meet such expenses. DB plan members obviously face the same risks but income stability means they are at least marginally better able to systematically plan for such costs, where such planning takes place.

This study examines the impact of anticipating the costs associated with age-related health treatment and aged-care services during the retirement phase on income level, income stability and longevity risk. To measure the impact of such costs on income sustainability and longevity, we simulate asset return data using historical bootstrap simulation to derive an optimal withdrawal income during retirement for a range of confidence levels. This allows us to test the sensitivity of income sustainability in relation to the retirement horizon, the magnitude and timing of health and aged-care costs, unexpected longevity and the interplay between risk aversion and asset allocation during retirement. We derive a series of ruin probability profiles that quantify the impact of both the timing and magnitude of health and aged-care costs on the safe withdrawal rate for a typical retirement portfolio.

Our analysis considers investors who either anticipate future health/aged-care costs or who fail to anticipate such future costs. The results establish a number of important outcomes related to the probability of investors outliving their retirement portfolio. First, we show that the greatest risk to income sustainability occurs when unexpected health costs translate into greater longevity. Paradoxically, this means that high costs associated with health treatment may result in a longer life however without a commensurate adjustment in asset allocation towards assets with a greater risk-return profile; it also risks premature wealth depletion. This is particularly true for risk-averse investors who bias their asset allocation towards low risk assets. Second, we show that the safe withdrawal rate is highly sensitive to the timing of health costs and moderately sensitive to later-life aged care costs. Third, we show that in a set of broad circumstances, the risk of premature wealth depletion can be mitigated through a type of dynamic lifecycle (DLC) strategy during the retirement phase.

2.0 Background

It is instructive to present an overview of the total number, the housing situation and the health situation of retirees in a representative country to better understand the magnitude of cost profile for later-in-life liabilities. In Australia, there are around 3.3 million people aged 65 years or older (referred to hereafter respectfully as ‘older people’) representing around 14% of the total population (ABS, 2012). The number of older people is expected to reach between 6.2 million and 7.9 million by 2050. This equates to around 25% of the population. It is important to note that around half of the current number of older people (7.5% of the total or 1.7 million people) also has a disability (ABS, 2012).

Using 2012 Australian Bureau of Statistics survey research, over 90% of older people live in a private dwelling (i.e. house, apartment or home unit) and nearly three quarters of these (71%) live with others. For people aged 80 years or more 77% lived in a private dwelling and over half (58%) are still living with others. In addition, 5.5% of older people were housed in cared-accommodation while 4.0% lived in 'other non-private dwellings' such as caravan parks and self-care units in retirement villages. These proportions of older people are likely to remain high with Australian Government policy regarding aged care pointing toward a greater emphasis on aging at home before advancing to residential aged care when greater medical intervention in required.

As people age their physical and mental functioning sometimes deteriorates and they become more susceptible to age-related conditions. Over 87% of older people report having a long-term health condition. This compares with 31% of people aged less than 65 years having a long-term health condition. Of older people who reported a long-term health condition, 93% are most affected by a physical condition and 7% are affected by a mental or behavioural disorder. These conditions range from arthritis (16%) and hypertension (11%) to back problems (9.4%). Even though the majority of older people live with others, there are around 60,000 people with a profound core activity limitation who live alone. Over 29% of older people need direct assistance with certain personal activities including health care (25%), mobility (18%), property maintenance (23%) and household chores (18%) (ABS, 2012).

Certain health conditions are chronic among older people. The prevalence of arthritis increases with age, from less than 1% of people aged under 25 years to 52% of people aged 75 years and over. Women are considerably more likely to have arthritis than men; at ages 75 years and over, around

60% of women have arthritis compared with 42% of men. The prevalence of cancer also increases with age, with 7% of people aged 75 years and over having cancer (compared with 1% of people aged 45-54 years). More men had diabetes than women (5% of men aged 2 years and over compared with 4% of women aged 2 years and over) and, as with many health conditions, the rate of diabetes increases with age. People aged 75-84 years had the highest rate of diabetes (17%). Lastly, it is well known that heart disease remains one of the leading causes of death worldwide, and the statistics in Australia are largely representative of this observation. The proportion of people with heart disease increases steadily with age, such that over one quarter (29%) of people aged 75 years and over had heart disease. The highest rate of heart disease is observed in men aged 85 years and over (47%).

The ABS surveys also reveal that the prevalence of some of the more costly health conditions dramatically increases with age. For example a person aged 80 years or over is seven times more likely to identify dementia or Alzheimer's disease as their main long-term health condition than someone aged 65 to 79 years (7.6% compared with 1%). In contrast, the proportion of those who reported arthritis as their main condition was similar across these age groups (17.3% compared with 15.9%). The trend appears to be that high-cost health condition treatment and care services needed to cope with the increased life expectancy of the population will predictably have a far greater impact on retirement wealth than the lower cost health services related to less acute age-related conditions.

In socio-demographic terms the proportion of people with heart disease and diabetes increases as the level of disadvantage increases. People living in areas of most disadvantage are more than twice as likely to have diabetes and heart disease compared with those living in areas of least disadvantage (8% compared with 3%). Over 75% of older Australians resided in a household with gross household income in the lowest two quintiles, while only 5.3% of older people were in the highest quintile of gross household income. There are social implications associated with this fact that lie beyond the scope of this analysis. However for a great proportion of retirees the burden of health and aged-care service costs will be incurred by them so planning for such liabilities is increasingly important.

Improvements in the treatment for certain illnesses such as cancer have accelerated in the past two decades. The survival rate for many cancers has increased by 30% over the last 20 years. For instance, in 2006 the 5-year survival rate (the percentage of patients that are alive five years after initial diagnosis) for the most common cancer in Australian women (breast cancer) was 88% and in 2010 the 5-year survival rate for the most common cancer in Australian men (prostate cancer) was 85% (AIWH, 2010). Such statistics cause us to sharpen our focus on retirement planning: if survival probabilities are rising for even those with the direst of health conditions then retirement planning becomes even more critical.

As people age their housing needs change. Changes could include modifying their existing home to accommodate ramps and rails or installing modest low-maintenance accommodation features. The change can also be more drastic when deciding to move to a smaller premise or to an aged cared facility (Productivity Commission, 2008). The most expensive place for older people to live is in residential aged care (Allen Consulting, 2002; 2007). In 2012 the average annual cost to government of a person in residential aged care was over $40,000 compared with costs of between $3,800-$7,000 for an older person who stays in their own home (depending on the level of care needed). In return for government subsidies that support home owners it is likely that retirees will be required to provide more for their retirement, including health care and aged care costs (Bruen, 2005; Yee, 2005). A comprehensive report by Grant Thornton in 2012 concluded that high-care aged facilities cost an average of $80,000 per bed (Ansell, et al. 2012).

In 2012, a government pension or allowance was the main source of income for two million older Australians (65%). People aged 65 years and over without disability were three times more likely to receive a wage or salary as their main source of income than those with disability (10.4% compared with 3.4%).

However fiscal constraints means that nation states (including Australia) are unlikely to be able to fully support an ageing population of retirees for 20-40 years' worth of pension payments. The need for liability-driven and goals-based investing has emerged for retirees to address all of their retirement needs, not only as a form of financial security at an individual level, but also as a form of prudent social policy. A goals-based approach focuses on funding personal financial goals and requirements rather than simply achieving higher investment returns relative to the market. Further, it designates an investment approach for a household based on their risk capacity rather than their risk tolerance. This approach is broadly similar to the approach used for asset-liability management in insurance companies and liability-driven investment strategies in pension funds. The approach is distinguished from these however in that it integrates financial planning and investment management to ensure that household goals (including health and aged-care services costs) are financed/funded efficiently (Fizel and Nunnikhoven, 1992). For a goals-based investing approach to be most efficient, all household assets and liabilities across a lifetime need to be considered. Assets represent the full set of resources available to the investor such as financial assets, real estate, employment income and social security. Liabilities represent all financial liabilities such as loans and mortgages, in addition to the capitalised value of the household's financial goals and aspirations.

For this approach to be successful, the required and/or desired income level in retirement needs to be articulated from the outset. The ultimate aim of this approach is therefore to guard against poor investment decisions by providing a clear process for identifying goals and choosing investment strategies for those goals. This approach not only adapts investment style to actual investors, it avoids the need to ensure that such investors have a superior understanding of financial markets and investment strategies.

3.0 Methodology

The success of a retirement portfolio in the presence of asset price volatility and liability uncertainty is a complicated problem in which the objective function cannot be evaluated precisely. When confronted with such issues, historical bootstrap simulation is widely accepted as a means of estimating the objective function by randomly generating values for uncertain outcomes from a known distribution of input variables.

3.1 Model worker

We illustrate the impact of later in life health and aged-care costs using the simple case of a typical female employee aged 50 who has made contributions to her pension plan throughout her working life which amounts to a modest $250,000 in superannuation.[4] She faces asset-return risk both during the accumulation phase and the retirement phase. This affects the value of her superannuation fund, given past and future contributions. We have specifically chosen a female investor because it underlines a key problem in retirement planning for many individuals; relatively low wealth coupled with a longer life expectancy.

We examine two aspects of her capacity to cater for health costs and aged-care costs; anticipated or expected cost occurrence and unanticipated or unexpected cost occurrence. We have chosen to work with annual returns in real terms.

The other key inputs for the representative investor is that her initial age is 50, retirement age is 65, investment horizon is to the age of 95, initial investment is $250,000, initial salary of $70,000, wage inflation is 2% pa, price inflation 2.5% pa, pension contribution rate 9.5% pa, tax 15%, and aged-care costs of $80,000 (aligned with the average cost of a high-care facility bed, see Ansell, et al. (2012)). We chose the high-care level of health/aged-care costs of $80,000 to represent the potential for

[4] In this context 'modest' refers to the absolute dollar value of the portfolio for a worker that has contributed for their entire working life. Compared to current actual female account balances, $250,000 is in fact quite high. For more on the gender-sensitive superannuation design, see Basu and Drew (2009b).

significant health issue affecting the investor from which portfolio recovery will be highly dependent on risk appetite. This level represents a cost impost of around 12% of her median portfolio value at the date of retirement.

3.2 Constant inflation adjusted withdrawals – stochastic optimisation

The model assumes that the retiree begins retirement with an initial withdrawal from their retirement portfolio and the post-withdrawal portfolio remainder is invested in stocks, bonds and cash. The portfolio earns an inflation-adjusted rate of return, weighted initially by a constant asset allocation, until the next annual withdrawal. A discrete time representation of the portfolio rate of return is

$$r_t^i = \sum_{j=1}^{n} w_{t,j} r_{t,j}^i, \quad (1)$$

where r_t^i is the weighted average portfolio return for simulation *i* at time *t*, $w_{t,j}$ is the portfolio proportion assigned to asset class *j* at time *t* and $r_{t,j}^i$ is the annual inflation-adjusted return for asset *j* at time *t* for simulation *i*. Ongoing withdrawals from the portfolio remain the same (in inflation-adjusted dollars), and the value of the portfolio is derived as

$$V_t^i = \left[V_{t-1}^i - MV_0\right]\left(1 + r_t^i\right), \quad (2)$$

where *V* represents the value of the portfolio and *M* is the constant withdrawal fraction amount.

We need to use stochastic optimisation in the model to identify the optimal withdrawal rate for a set of asset allocations and a known investment horizon that minimises the probability of portfolio ruin. We use the stochastic optimisation process for three cases; optimal withdrawal rates in the absence of health and aged-care liabilities, optimal withdrawal rates in the presence of expected health and aged-care liabilities and optimal withdrawal rates after the occurrence of unexpected health and aged-care liabilities.

Prior to retirement we incorporate annual cash flows into the accumulation account up to the nominated date of retirement as well as initial portfolio conditions. The portfolio value V_t at time *t* is defined as

$$V_t = (V_{t-1} + CF_{t-1})(1 + X_t) - LS_\tau + 1_E(SSP_{\tau<T}); \ \ t, \tau < T, \quad (3)$$

where CF_t is the after-tax cash inflow (positive) or outflow (negative), X_t is the weighted average portfolio return $w_n' r_n$ at time *t*, LS_τ is any lump sum payment withdrawn at retirement date τ and $1_E(SSP_{t>\tau})$ is an indicator function where 1_E is equal to one if the investor qualifies for social security payments (*SSP*) during retirement $t > \tau$ and zero if the investor does not qualify for such payments. Both the retirement date τ and the withdrawal dates *t* are assumed to be less than the terminal date *T* for all payments as selected by the investor. The value V_t of the portfolio at *t=0* is set to the initial portfolio value of the investor.

In contrast with deterministic approaches to retirement planning, where both the investment horizon and the investment return are assumed to be known with certainty, in this analysis we represent the variables as stochastic. We derive the stochastic present value at either the date of retirement (which assumes a deterministic terminal portfolio value) or at any point before retirement as

$$\widehat{PV} = \sum_{i=1}^{\tilde{T}} \prod_{j=1}^{i} \left(1 + \tilde{r}_j\right)^{-1}, \quad (4)$$

where $\tilde{T}$ is the random time of death (in years) and $\tilde{r}_j$ is the random investment return in year *j*. As $\tilde{T} \to \infty$ the stochastic PV simply reduces to the infinitely-lived endowment (Milevsky, 2006). The frequency of the above measure can be reduced to quarters or months as required without loss of generality.

The simulation process in this model assumes $\tilde{T}$ is fixed and is estimated by the investor. This greatly simplifies the simulation and then optimisation process.

The asset values and projections are simulated 10,000 times and the key percentiles at each time *t* are estimated from the simulation. A range of percentiles are extracted from the simulated terminal values (at time *T*) for the investor's portfolio and then used as the future value to iterate backwards to retirement date τ. To conduct the search we use a simple generalised reduced gradient search algorithm (Lasdon et al. 1978) to solve for the annual withdrawal over the withdrawal period ($\tau \rightarrow T$), which is also simulated 10,000 times to achieve convergence. This method is sufficiently robust to find at least a local optimum where the function is continuously differentiable. This approach is also known to be robust relative to other nonlinear optimisation methods.

The algorithm needs input function values as well as the Jacobian, which we do not assume to be constant for our nonlinear model. We approximate the Jacobian using finite differences re-evaluated at the commencement of each major iteration (i.e. the major percentile terminal values).

The investor has the choice to alter the risk of the portfolio (through asset allocation). For this model we assume three asset classes (stocks, bonds and cash) and across five broad sets of asset allocations that represent relative levels of risk aversion. The weightings for each category are provided in Table 1.

Risk	**Stocks**	**Bonds**	**Cash**
Very high	90%	10%	0%
Moderate	50%	40%	10%
Balanced	40%	40%	20%
Conservative	30%	40%	30%
Very low	10%	30%	60%

Table 1: Asset class weights for 10 levels of risk aversion.

A simulation of 10,000 iterations generates a single probability of ruin for a given portfolio allocation, age at retirement, stochastic inflation-adjusted portfolio return, deterministic occurrence of death and a fixed stochastically-optimised withdrawal rate. Each set of simulations is conducted to derive the impact on withdrawal rates and the probability of ruin for the three cases:

1. optimal withdrawal rates in the absence of health and aged-care liabilities,
2. optimal withdrawal rates in the presence of expected health and aged-care liabilities, and
3. optimal withdrawal rates after the occurrence of unexpected health and aged-care liabilities.

To solve for the optimal withdrawal rate we use the complex method of constrained optimization first proposed by Box (1965) and then improved by Guin (1968). This approach is capable of optimising a complex objective function with few constraints on the optimisation function itself while also avoiding the need to explicitly compute the derivatives of the function itself. Studies by Stout and Mitchell (2006) and Stout (2008) have used a similar algorithm to identify optimal withdrawals for a narrower suite of input parameters. From the results we will be able to better understand both the optimal investment strategy and the optimal withdrawal rate when significant health and aged-care liabilities are taken into consideration.

This optimisation methodology can be more simply demonstrated using a diagram. Figure 1 shows that the simulation estimates the range of outcomes available to an investor through both the accumulation and retirement phase. The stochastic optimisation process aims to select a constant withdrawal rate through the retirement phase that yields an expected terminal wealth of zero at the 5% confidence level coinciding with the investor's 'expiry' date (death or other nominated future date).

The Box Method iteratively searches possible input values for withdrawal amounts to reduce the simulated probability of ruin at a 5% confidence level, to find a global minimum solution (if one exists). The optimal withdrawal values are then used in a second set of block bootstrap simulations to estimate the probability of portfolio ruin.

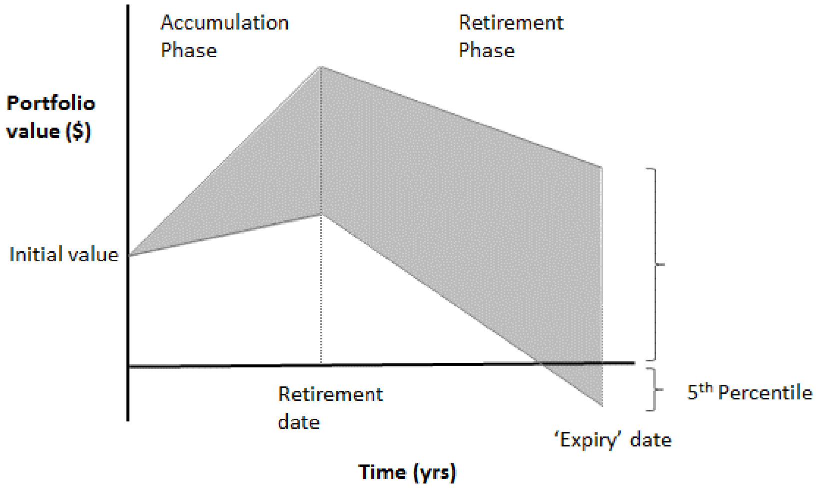

Figure 1: Simulation process for investor estimating a fixed withdrawal rate leading to terminal wealth depleting to zero at the 5th percentile.

Ultimately the model is able to answer the basic question: at what level can an investor set their retirement income expectations and expenditure levels? This motivates the investor to focus on the almost certain income level which we set to a confidence level of 5%, and avoids setting the objective function to simple maximise wealth at the date of retirement and then hoping the portfolio value is sufficient so that the investor does not outlast their portfolio. Indeed, the intention of goals-based investing is to match the time-weighted value of assets and liabilities that cater for cash flows through an investor's working life as well as through retirement.

4.0 Data and calibration

Asset class return data for the historical bootstrap model were obtained from Global Financial Data (GFD). The S&P/ASX 200 Accumulation Index (in AUD) return series is used to represent Australian stocks. This index uses Lamberton's indices for the Australian Stock market from 1882 to 1958, Sydney's All-Share indices from 1958 to 1971, the Statex Accumulation Index from 1971 to 1979, the ASX All-Ordinaries Index from 1979 to May 1992 and the S&P/ASX 200 Accumulation Index from June 1992 to December 2013. Prior to 1971, the total return was calculated based upon price indices and dividend yield data for the Australian Stock Exchange. The 10-year Government Bond Return Index (in AUD) returns series was used to represent Australian bond data. This data was obtained from The Economist for 1858-1931, D. McL. Lamberton, 'Security Prices and Yields, Part III,' Sydney Stock Exchange Official Gazette (December 15, 1958, p. 556) for 1875-1925, the League of Nations Statistical Yearbook (Geneva: League of Nations) for 1926-45, the New South Wales Statistical Register for 1946-1956 and the Reserve Bank of Australia (Monthly Statistical Bulletin) for 1956-2013. Total Returns Bills Index (in AUD) is used to represent Australian cash returns. This data was obtained from The Economist for 1858-1931, D. McL. Lamberton, 'Security Prices and Yields, Part III,' Sydney Stock Exchange Official Gazette (December 15, 1958, p. 556) for 1875-1925, the League of Nations for 1926-45 and the Reserve Bank of Australia, Statistical Bulletin for 1970-2013. The bill index uses the bank deposit rate from 1834 until June 1928 and Treasury bill yields thereafter. We collated and synchronised the data to derive a series of annual returns from October 1882 to December 2013. The summary statistics for the annual data is provided in Table 2.

Long-term assets exhibit mean reversion, there is a positive long-run equity risk premium, most assets exhibit leptokurtosis and the contemporaneous correlation between financial asset returns and real earnings growth is not strong. We also find evidence that the real yield on T-bills exhibits some degree of persistence over time however when we measure serial correlation at a yearly frequency most of the persistence disappears.

	Australian equities	Australian bonds	Australian cash
Mean	12.12%	5.88%	4.20%
Stand Dev	13.03%	7.86%	1.00%
Skew	-0.24	0.17	0.51
Kurt	4.17	4.14	3.27
JB-Stat	312	278	222
P-value	0.00	0.00	0.00

Table 2: Summary statistics for annual return series (linear) of Australian stocks, foreign stocks, Australian bonds and Australian bills, October 1882 – December 2013.

5.0 Results

5.1 Anticipated health and aged-care costs

The objective function of the model is to maximise the annual withdrawal of income subject to the constraint that the probability of ruin is minimised over the expected life of the investor. In the case where investors anticipate some form of cost requirement to finance health and/or aged care costs at some point during their retirement, investors will naturally ease back on their withdrawals so that there are sufficient funds in their portfolio to both pay the discrete cost and fund the remainder of their retirement. Therefore the objective function we employ here takes into consideration the need for an investor to withstand a single $80,000 discrete payment at some point during retirement.

Table 3 provides the optimal withdrawal rates computed as the 5th percentile of the median (expected) portfolio value at the date of retirement with anticipated health and/or aged-care costs of $80,000 due at any point, for three life expectancies. For example, an investor who is relatively healthy and expects to live to 90 years of age with retirement savings invested in a balanced portfolio, and expecting to pay a liability of $80,000 at any point during retirement, will optimally withdraw 4.28% of their portfolio value at the date of retirement each year. This equates to $24,751 pa.

Life expectancy	Portfolio	Withdrawal rate	Withdrawal $	% of unexpected withdrawals
80	Very high	3.83%	36,250	92.66%
	Moderate	5.16%	32,430	83.96%
	Balanced	5.05%	32,423	86.82%
	Conservative	5.00%	29,887	82.44%
	Very low	5.04%	27,501	94.36%
90	Very high	2.92%	27,639	90.71%
	Moderate	4.16%	27,501	92.63%
	Balanced	4.28%	24,751	89.56%
	Conservative	3.68%	21,015	80.12%
	Very low	3.43%	18,750	94.89%
100	Very high	2.90%	22,778	82.41%
	Moderate	3.50%	22,901	84.09%
	Balanced	3.15%	21,821	80.88%
	Conservative	3.14%	18,750	88.11%
	Very low	2.63%	14,376	86.80%

Table 3: 5th percentile annual optimal withdrawal rates for each of the five asset allocation portfolios when anticipating health / aged-care costs assuming a given life expectancy.

Figure 2 depicts the probability of ruin profiles for our investor who incurs $80,000 in health and/or aged-care costs at a given point during retirement, and lives to the age of 80. These estimates were obtained for a range of five asset allocations – very high, moderate, balanced, conservative and very low – as outlined in Table 1. For instance, the probability of portfolio ruin for the investor who incurs health and aged-care costs at age 65 with a constant asset allocation to a moderate portfolio is around 7%. All of the asset allocations result in broadly similar ruin profiles for this short horizon, and the ruin probability is not very sensitive to risk preference. It is necessary to account for a range of asset allocation strategies to cover the range of risk preferences most investors are able to tolerate.

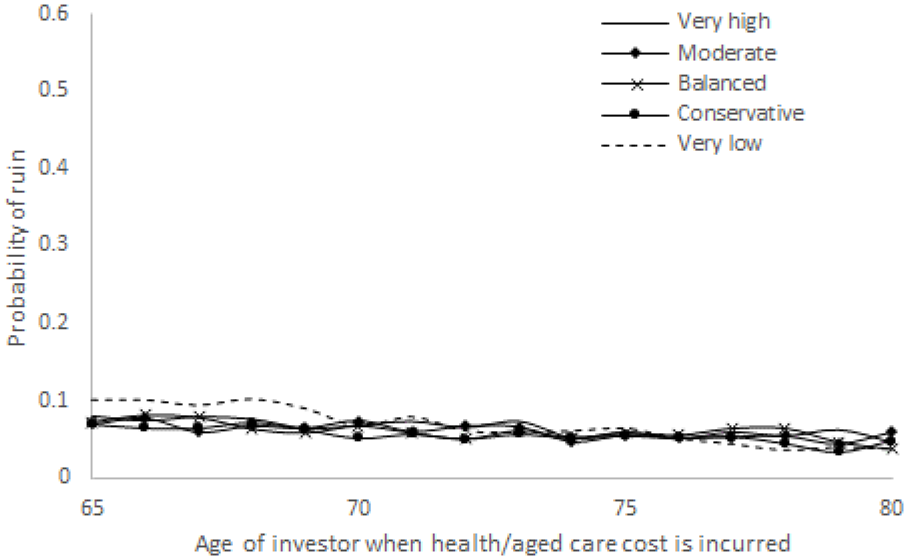

Figure 2: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with known health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 80, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000. Optimal withdrawal rates are used.

Figures 3 and 4 depict the same optimisation process and ruin profiles for an investor who looks to extend their retirement horizon to 90 years of age and 100 years of age respectively. The probability of ruin for all asset allocations declines relative to the 80 years of age horizon because the investor lowers their spending rate during retirement to cater for an expected health and/or aged care cost liability during retirement, which is highlighted in Table 3. Additionally, each portfolio has sufficient time to recover from a cost liability such that the probability of ruin is either stable or gradually declines.

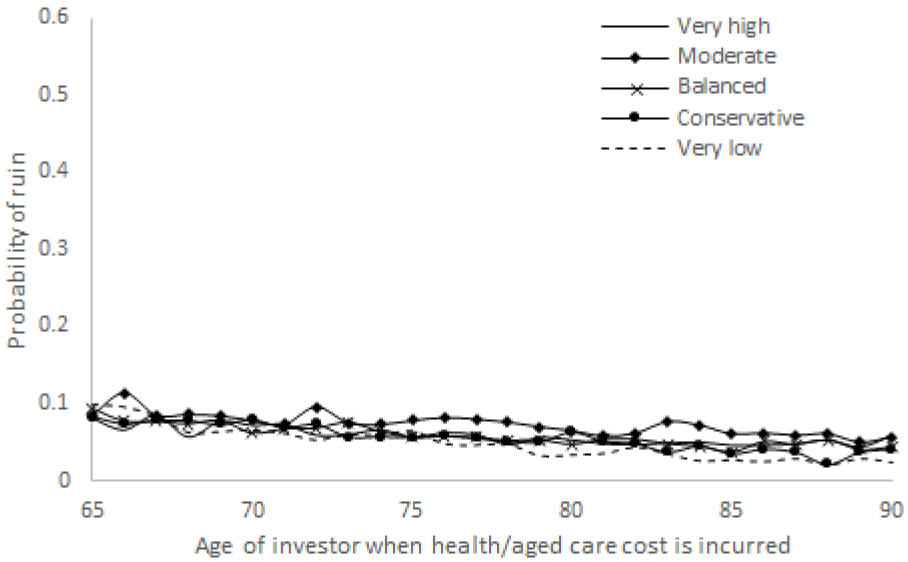

Figure 3: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with known health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 90, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000. Optimal withdrawal rates are used.

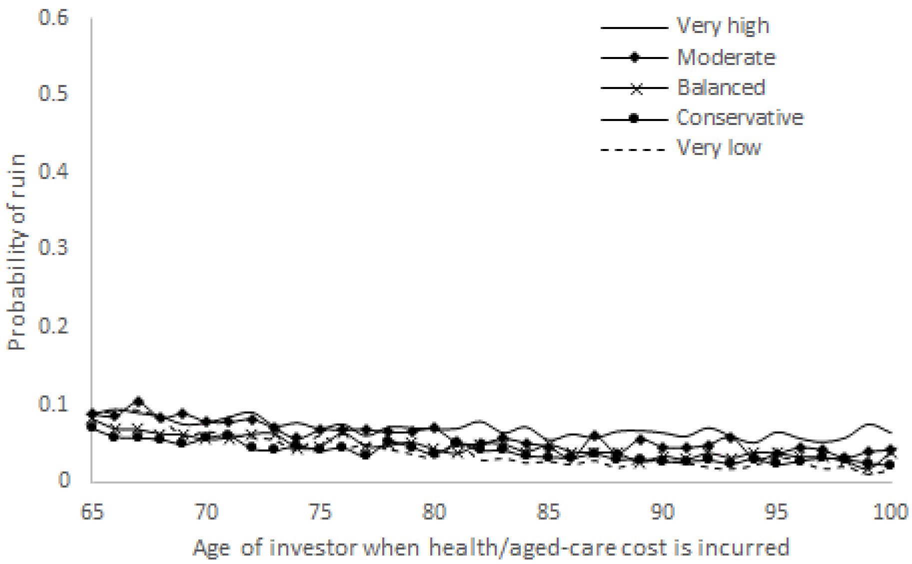

Figure 4: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with known health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 100, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000. Optimal withdrawal rates are used.

If we use the ASFA Retirement Standard Modest lifestyle for a single person of $23,363 p.a. expecting to live to 90 years of age we observe the probability of ruin profiles in Figure 5. Similarly if we use the ASFA Retirement Standard Modest lifestyle for a couple of $33,664 p.a. we observe the probability of ruin profiles in Figure 6.

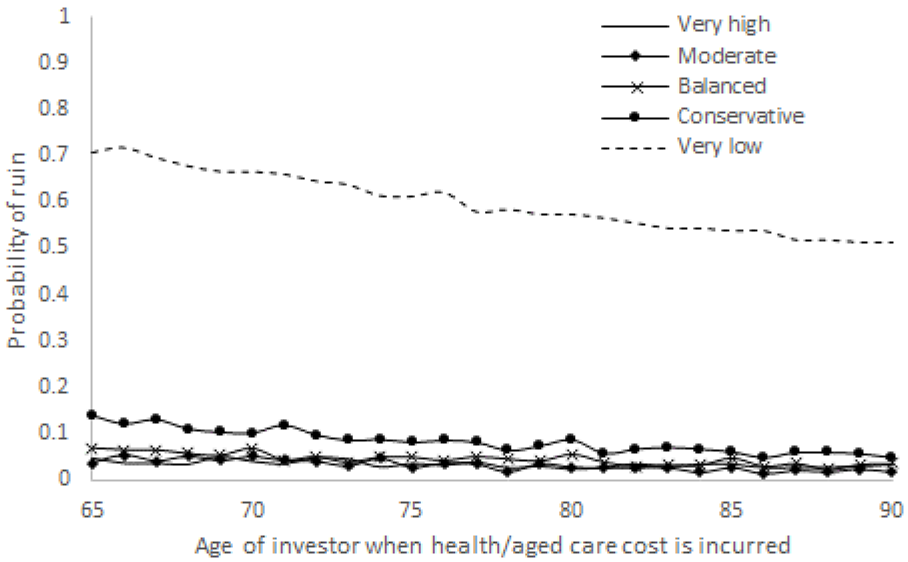

Figure 5: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with known health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 90, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000. Withdrawal rate of $23,363 pa used (ASFA Retirement Standard Modest lifestyle for a single person).

At a marginally higher withdrawal rate, investors incurring significant health and/or aged-care costs will experience a potentially higher probability of ruin early during the retirement phase if assets are too conservatively invested. As shown in Figure 5, ruin is very high for low risk portfolios while higher risk portfolios that are heavily weighted towards stocks exhibit a low and declining probability of ruin through the retirement phase. At a significantly higher withdrawal rate however, investors incurring significant health and/or aged-care costs will experience a potentially higher probability of ruin early during the retirement phase if assets are invested in towards conservative portfolios. As shown in Figure 6, ruin is almost certain for very low risk portfolios while higher risk portfolios that are heavily weighted in stocks exhibit a declining probability of ruin through the retirement phase. Indeed higher risk portfolios are dominant against portfolios containing a declining risk profile.

For higher withdrawal rates, when incurring significant health and/or aged-care costs the probability of ruin is generally directly related the level of risk implicit in the asset allocation. As shown in Figure 6, ruin is almost certain for low risk portfolios while higher risk portfolios that are heavily weighted in stocks, the probability of ruin is significantly less and some degree of portfolio recovery is possible.

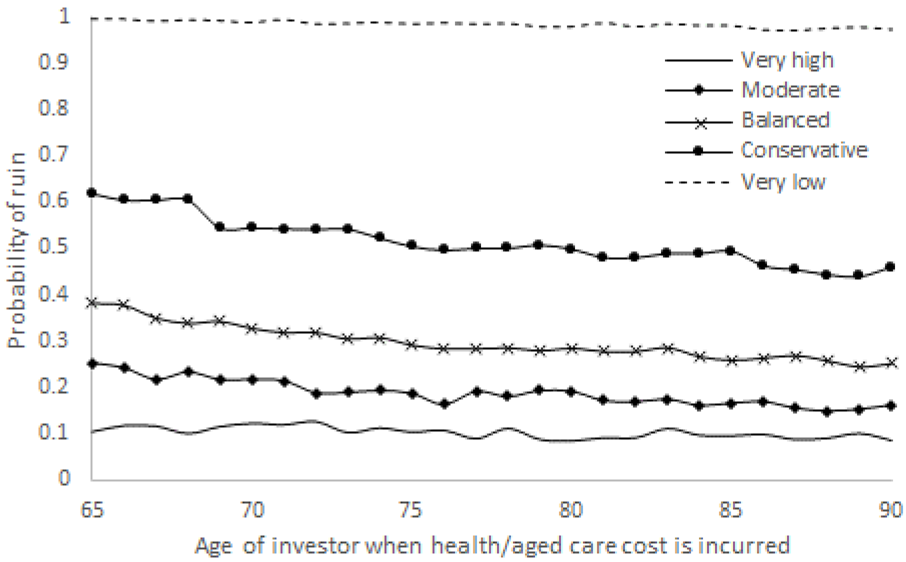

Figure 6: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with known health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 90, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000. Withdrawal rate of $33,664 pa used (ASFA Retirement Standard Modest lifestyle for a couple).

5.2 Unanticipated health and aged-care costs

We now consider the same analysis but instead, we do it for an investor who fails to anticipate any form of health or age-care costs to occur during their retirement. In this case the investor optimises their withdrawal rate based on an expected retirement horizon without any consideration made for discrete adverse portfolio events. Subsequent to the event however, the investor then needs to re-optimise their withdrawal rate based on the same expected retirement horizon. We then calculate the probability of ruin for this investor over the same five asset allocations as in the anticipated cost case study above. The only difference is that the investor does not adjust their optimal withdrawal rate to account for the possible occurrence of an $80,000 cost for health and aged-care costs at some stage during retirement.

Life expectancy	Portfolio	Withdrawal rate	Withdrawal $
80	Very high	4.13%	39,121
	Moderate	5.75%	38,626
	Balanced	5.89%	37,344
	Conservative	6.19%	36,251
	Very low	6.24%	29,145
90	Very high	3.22%	30,470
	Moderate	4.20%	29,688
	Balanced	4.36%	27,637
	Conservative	4.70%	26,230
	Very low	4.23%	19,759
100	Very high	2.90%	27,640
	Moderate	3.89%	27,234
	Balanced	4.34%	26,981
	Conservative	3.64%	21,280
	Very low	3.55%	16,563

Table 4: 5^{th} percentile annual optimal withdrawal rates for each of the five asset allocation portfolios without anticipating health / aged-care costs assuming a given life expectancy.

Table 4, which is of the same format as Table 3, provides the optimal withdrawal rates computed as the 5^{th} percentile of the median (expected) portfolio value at the date of retirement with no anticipated health and/or aged-care costs at any point, for three life expectancies. For example, an investor who is relatively healthy and expects to live to 90 years of age with retirement savings invested in a balanced portfolio, and does not expect to pay any health/aged-care costs during retirement, will optimally withdraw around 4.36% of their portfolio value at the date of retirement each year. This equates to around $27,637 pa (and thus exceeds the ASFA Retirement Standard Modest lifestyle for a single person of $23,363 pa).

As shown in Figure 7, relative to the ruin profiles in Figure 2 of less than 10%, the probability of ruin for an investor with an investment horizon to 80 years of age dramatically increases for each of the five asset allocation strategies. The probability of ruin is higher for portfolio allocations that are weighted towards bonds and cash. The profiles fluctuate around a central trend which is an artefact of the bootstrap simulation process using historical data. The profiles have not been approximates using trend analysis: rather, we retain the raw results to avoid approximations.

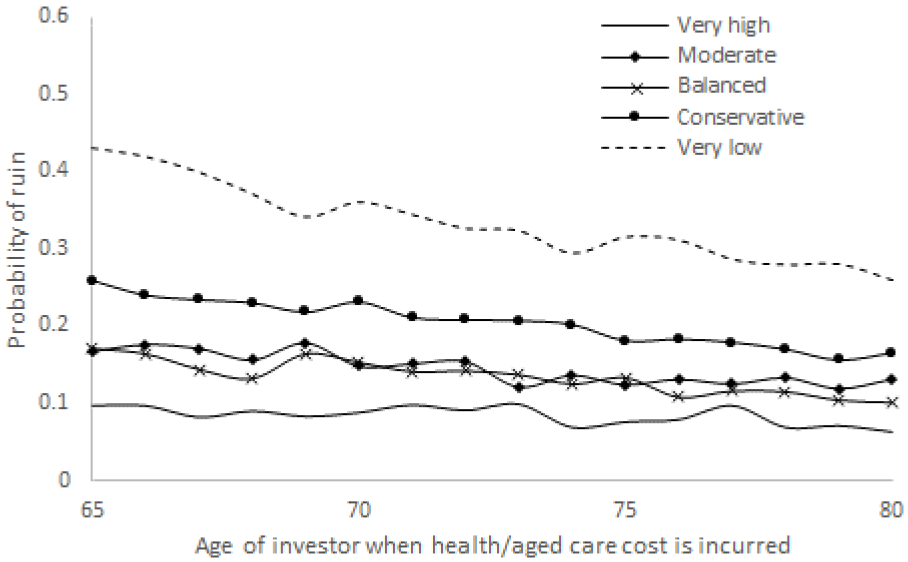

Figure 7: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with unanticipated health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 80, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000 incurred at each age. The probability of ruin represents the probability of depleting the retirement portfolio given an unexpected health/aged-care cost liability of $80,000 incurred at a particular year and the investor continues to live until 80 years of age. Optimal withdrawal rates for each asset allocation (based on risk tolerance) are used.

Figures 8 and 9 depict the same optimisation process and ruin profiles for an investor who looks to extend their retirement horizon to 90 years of age and 100 years of age respectively. In contrast to investors who anticipate significant heath and aged-care costs, the probability of ruin for all asset allocations actually increases because the investor fails to adjust their spending rate during retirement to cater for an expected health and/or aged care cost liability. It should be noted that as the investment horizon increases, only do the less conservative portfolios (very high, moderate and balanced) eventually recover from the cost liability such that the probability of ruin eventually converges to a value that is near the ruin probabilities predicted for the investor who anticipates such costs.

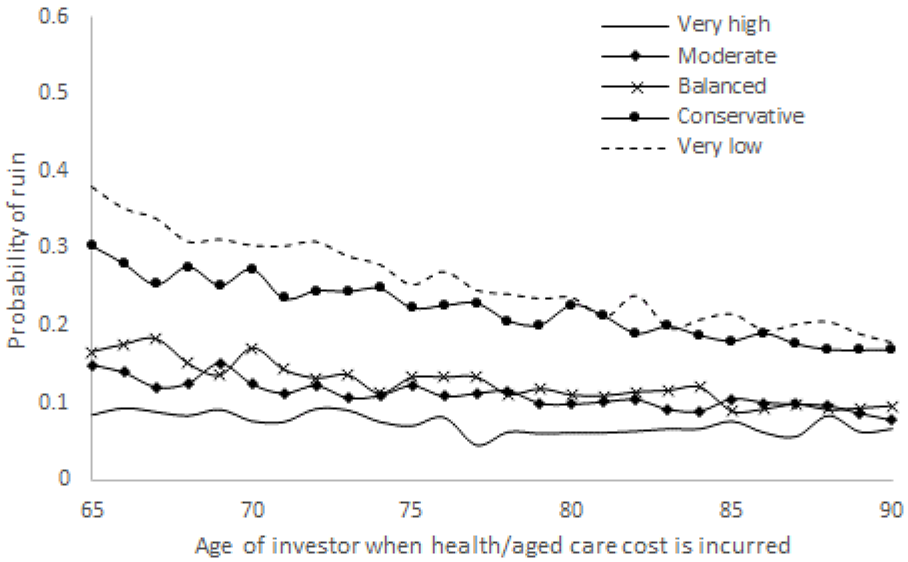

Figure 8: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with unanticipated health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 90, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000 incurred at each age. The probability of ruin represents the probability of depleting the retirement portfolio given an unexpected health/aged-care cost liability of $80,000 incurred at a particular year and the investor continues to live until 90 years of age. Optimal withdrawal rates for each asset allocation (based on risk tolerance) are used.

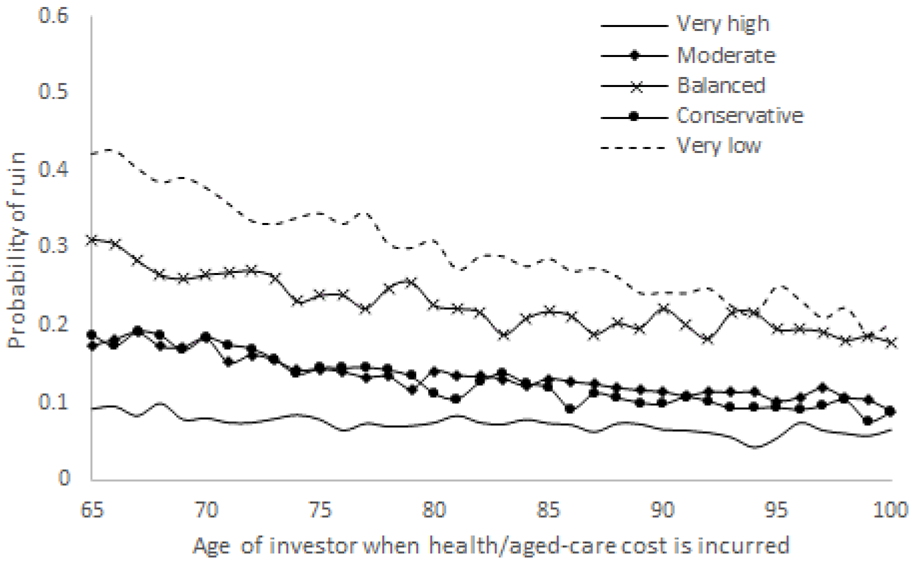

Figure 9: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with unanticipated health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 100, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000 incurred at each age. The probability of ruin represents the probability of depleting the retirement portfolio given an unexpected health/aged-care cost liability of $80,000 incurred at a particular year and the investor continues to live until 100 years of age. Optimal withdrawal rates for each asset allocation (based on risk tolerance) are used.

This outcome suggests that the significant decline in the portfolio value after incurring an unexpected health or aged-care cost liability increases the probability of ruin when the portfolio is heavily weighted towards low risk assets. At the other extreme of the asset allocation continuum however, beyond a certain point where the portfolio is allowed to recover, the probability of ruin for higher-risk portfolio strategies plateaus or declines. The lower risk strategies that are weighted towards bonds and cash do not have sufficient time for the portfolio to recover after an unexpected liability, with the investor drawing a modest income. Lower-risk investment strategies will inevitably lead to a higher probability of ruin for longer investment horizons.

So a higher risk investment strategy through the decumulation phase appears to dominate the optimal investment approach for investors who incur a significant health or aged-care cost liability at some point during retirement, particularly when examining the investment behaviour over long time horizons. For investors with a higher chance of survival beyond their life expectancy however who have a low tolerance to risk, are there a mixture of strategies that can cater for their need to experience lower volatility while simultaneously reducing the probability of ruin? An approach modelled on the dynamic lifecycle investment philosophy that obtains best of both worlds may be possible.

5.3 Dynamic lifecycle approach to recover from unanticipated costs

It is fair to say that a great number of investors will fail to fully anticipate significant age-related health and/or aged-care costs during their retirement, and as such will optimise their spending pattern to align with their portfolio level and life expectancy. Many investors will defer to the State to make up the shortfall in health and aged-care costs.

However as social policy reform shifts the responsibilities for age-related costs to individuals, it is clear that the State will soon be unable to make up the entire difference for such costs. To account for the cost gap confronting an investor who fails to anticipate any form of health or age-care costs during retirement, a key question is whether some form of dynamic asset allocation strategy during both the accumulation and retirement phases can remedy the investor's portfolio depletion to sufficiently recover withdrawal rates to the same value as if the health and aged-care costs were in fact anticipated.

Dynamic asset allocation strategies have been shown to minimise the effects of sequencing risk during the accumulation phase. Basu, Byrne and Drew (2011), Pfau and Kitces (2013) and Ang, Chen and Sundaresan (2013) advocate increasing equity allocations if a retiree is falling short of their retirement goals, in terms of maximising wealth at the date of retirement. They show that there is a greater chance of a successful retirement even if retirees experience an unfavourable market environment early, because a rising equity allocation through time will maximise their exposure when the market rebounds. Unfortunately, this dynamic asset allocation strategy requires a higher level of risk tolerance from retirees. However a product-centric strategy of higher risk equity funds in combination with a dynamic asset allocation approach can make such a strategy more tolerable for conservative retirees.

During the accumulation phase, portfolio adequacy based on a defined terminal wealth target can be optimally achieved using target-driven asset allocation strategies such as a dynamic lifecycle strategy (DLC) strategy. The DLC strategy increases the allocation to riskier asset classes when workers' portfolio wealth is less than a defined adequacy target. The glide-path of a DLC strategy is not pre-determined because the asset allocation policy is not only dependent on a worker's retirement date but also on the performance of the portfolio relative to a retirement target. When the portfolio wealth is greater than an adequacy target the allocation shifts towards more defensive assets, and when wealth falls below the target the portfolio shifts its weight towards growth assets. The DLC strategy is a flexible approach that preserves terminal wealth as the primary objective, particularly in the presence of sequencing risk. This approach is in sharp contrast to the static and deterministic allocation strategies of target risk funds (TRFs) and target date funds (TDFs) that subordinate terminal wealth to a secondary aim behind maintaining a pre-determined policy portfolio.

The same approach can also be deployed during the retirement phase to preserve portfolio wealth with the constraints around a minimum withdrawal rate along with minimising the probability of ruin over the investment horizon. These two competing constraints can be reconciled through the dynamic optimisation approach discussed above, only with the added constraint regarding year-by-year ruin probability minimisation.

The 'drawdown dynamic lifecycle strategy' (DDLC) is as follows. The DDLC strategy is partitioned into three investment periods. First, for the years leading to the occurrence of an unanticipated health or aged-care cost liability (an event such as high needs care or high pharmaceutical costs), the strategy is heavily weighted toward high risk assets so that Australian stocks dominate 90% of the portfolio. The rationale for the initial allocation to growth assets only is that the objective of the investor is to continue to maximise wealth over the first 10 or so years of their retirement horizon. Consistent with lifecycle theory, the investor (now a retiree) should have sufficient time to recover wealth over this period if stock market performances have been unfavourable. Second, when the retiree enters the higher risk zone for incurring health or aged-care costs (beyond the age of 75) and then incurs a significant cost, the DDLC strategy switches to a second investment period, which is 10 years in length or until the 'expiry' of the retiree, whichever is sooner. Third, the remaining partition extends from the second partition (i.e. 20 years since retirement) out to the date of 'expiry' of the investor. Each of the three partitions have different asset allocation rules.

We examine three DDLC strategies, each differing only by the proportion assigned to growth and defensive assets. For the second and third partitions in each of the three strategies, the below-target ruin probability is 100% in growth assets. The above-target ruin probability in the second partition and the third and final partition is provided in Table 5.

	Below target portfolio	**2nd partition above target portfolio**	**3rd partition above target portfolio**
DDLC 1	100% Growth 0% Defensive	60% Growth 40% Defensive	40% Growth 60% Defensive
DDLC 2	100% Growth 0% Defensive	80% Growth 20% Defensive	20% Growth 80% Defensive
DDLC 3	100% Growth 0% Defensive	50% Growth 50% Defensive	20% Growth 80% Defensive

Table 5: Drawdown Dynamic Lifecycle (DDLC) strategy definitions.

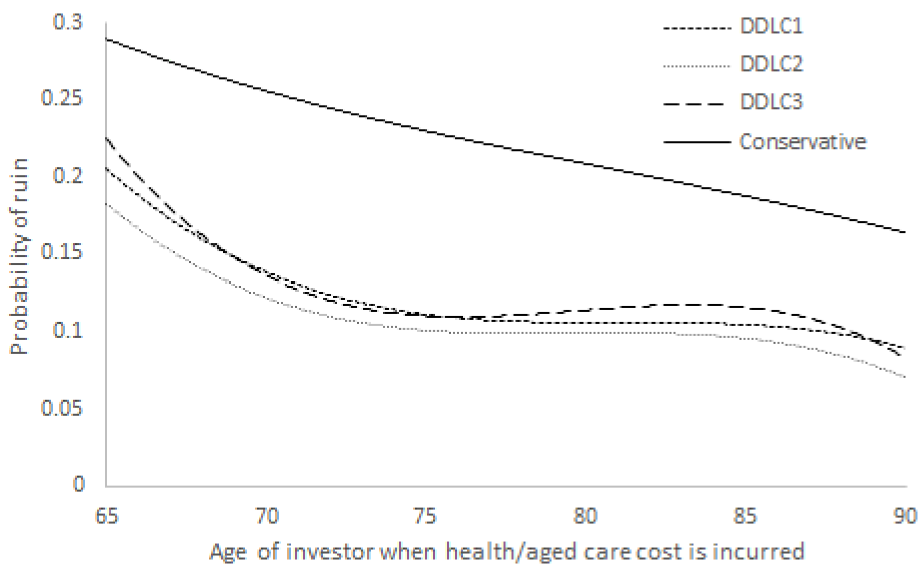

Figure 10: Probability of ruin for an optimised withdrawal rate for a range of Drawdown Dynamic Lifecycle (DDLC) asset allocations with unanticipated health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 90, initial investment of $250,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000. Optimal withdrawal rate for an investor initially allocated to a 'conservative' portfolio is used.

The rationale for the increasing proportion of growth assets in the above-target ruin probabilities in each of the final two partitions is to reduce or at least stabilise the probability of ruin as the investment horizon stretches towards the investor's expiry date, so long as the withdrawal rate remains above the originally derived rate. Unlike other common asset allocation strategies, the DLC strategy uses performance feedback to control the asset allocation at any point in time.

All three DDLC strategies outperform the static conservative strategy in terms of minimising the probability of ruin after incurring an unexpectedly large health / aged-care related cost. More specifically, the DDLC 2 strategy outperforms both the DDLC 1 and DDLC 3 strategies. The three DDLC strategies unsurprisingly converges to the constant conservative strategy probability of ruin profile after around 20 years of retirement. The DDLC 2 strategy is a more aggressive version of the other two DDLC strategies and it invests heavily in growth assets during the period of highest vulnerability for retirees. In the 75-85 years of age period, the imposition of a significant cost will impact heavily on longevity risk and without an aggressive portfolio recovery plan, the probability of ruin will remain very high (e.g. greater than 20% for a conservative investor). A probability of ruin of over 20% each and every year of retirement after a one-off health / aged-care cost burden could become psychologically debilitating. Preparing a dynamic recovery plan like the ones we have tested here can at least halve that probability and ensure that longevity risk is more manageable.

These results highlight that a DDLC-style approach can augment both portfolio recovery and minimise ruin probability over a long horizon in event of health and/or aged-care costs, in the same way as in the accumulation phase, for a typical investor. It is important to note that the median values for higher risk strategies will obviously dominate lower risk strategies. But as shown in the analysis above, when stabilising the probability of ruin becomes a major constraint, then the capacity to dynamically adjust investment strategies in response to this constraint and to the need to maintain a minimum income level above the ASFA Retirement Standard Modest lifestyle level.

6.0 Discussion

The stochastic optimisation model also allows for age pensions, lump sum withdrawals and accessing residential housing stock which are common cash flows that affect portfolio wealth and the probability of ruin. These additions to the model augment the range of investment strategies available, including DDLC strategies as discussed above.

6.1 Incorporating the age pension

A great number of retail investors will continue to rely on the age pension to supplement retirement income. This is included in our calculations where income and asset means tests are met. The age pension however is implicitly incorporated into the model via the *SSP* variable in Equation (3).

6.2 Lump sum withdrawals at the date of retirement

The model allows for lump sum withdrawals on the date of retirement which provides greater flexibility for investors to gauge the implications of extinguishing mortgages and other loans that will deplete monthly earnings. The lump sum withdrawal is assumed to be tax exempt.

6.3 Sensitivity of results to initial investment portfolio value

Clearly the probability of ruin is sensitive to the initial portfolio values used for the simulation. For instance, to demonstrate the degree of sensitivity, if we use the ASFA Retirement Standard Modest lifestyle for a couple of $33,664 p.a. with an initial portfolio balance that is twice the amount used in our initial model ($500,000) we observe the updated probability of ruin profiles in Figure 11.

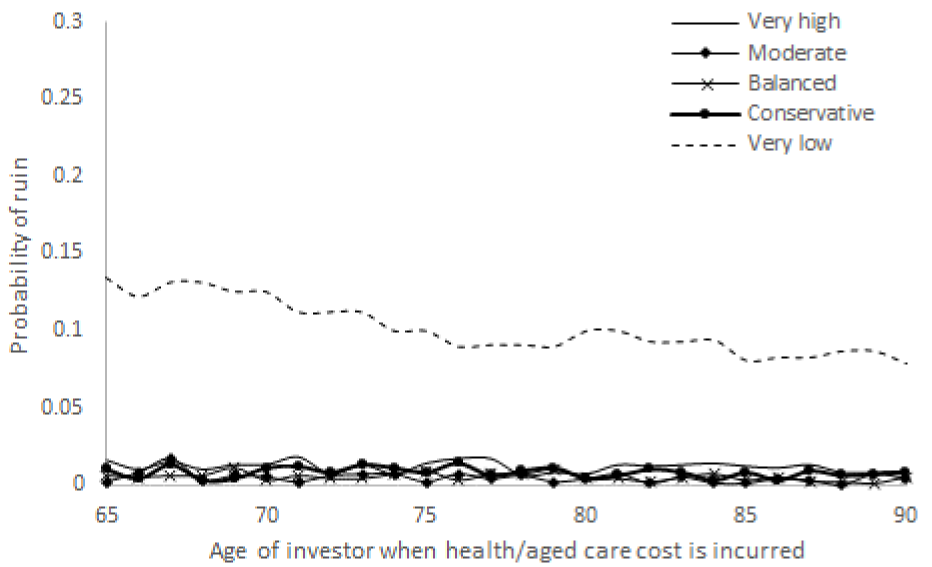

Figure 11: Probability of ruin for an optimised withdrawal rate for a range of asset allocations with known health and aged-care costs incurred at each age. Investor initial age 50, retirement age 65, investment horizon age 90, initial investment of $500,000, salary $70,000, wage inflation 2%, price inflation 2.5%, pension contribution rate 9.5%, tax 15% and health/aged-care costs of $80,000. Withdrawal rate of $33,664 pa used (ASFA Retirement Standard Modest lifestyle for a couple).

The probability of ruin declines significantly with the high risk asset allocation strategy attracting the greatest ruin profile over the investment horizon. Clearly initial portfolio value is the key driver for reducing the probability of ruin, however the degree of sensitivity is quite significant.

6.4 Accessing housing stock wealth

The provision of funding through retirement may be augmented by accessing housing wealth. Retirees can monetise their residential home in a number of ways. First, they can downgrade their house to a less expensive or rental home to access at least some of the money from their property (McNelis, 2007). Second, the retiree can adopt a ‘sell and stay’ home reversion model where they sell their residential home for an amount less than market value, but retain the right to continue living in the dwelling until they move out or die. Third, an alternative to the previous approach is the ‘stay and not sell’ model which involves the retiree taking out an additional loan (such as a second mortgage or a reverse annuity mortgage) allowing the retiree to borrow cash against the value of their home that is repaid with interest when the house is eventually sold (from the estate of the retiree).

Bridge et al. (2010) surveyed 192 brokers who dealt with reverse mortgages and reported that 72% of clients were aged between 60 and 74 years while only 28% were aged 75 years or older. The trend is for many ‘younger’ clients (60-74 years) to extract money out of their homes early in their retirement only to find that they have insufficient equity remaining in their home to afford a bond for residential care when they’re much older (beyond 85 years of age).

The results for the probability of ruin do not explicitly include the option to access residential housing wealth and so our predictive results provided above may be more conservative than what is observed.

7.0 Conclusion

The stochastic optimisation / dynamic goal-oriented investment methodology has a number of attractive features:

- The model is extremely flexible and can accommodate almost any set of assumptions or features relating to existing types of pension arrangements. The model therefore has considerable practical potential.
- The methodology allows us to develop sensitivity and ‘what if?’ experiments by changing key assumptions and observing how these changes affect our results. These exercises are obviously useful because they identify the key factors affecting results and gauge the response to particular assumptions.
- The model is naturally extended beyond the accumulation phase (the period up to retirement) to deal with the distribution (or post-retirement) phase. This is a necessary element of retirement modelling that has historically been disaggregated from accumulation phase modelling by retirement planning scholars.

We examined the probability of ruin for a range of investment strategies for investors who face expected and unexpected health and aged-care costs during retirement. Broadly, investors who anticipate health and aged-care costs suffer lower probability of ruin over the retirement horizon compared with investors who fail to account for such liabilities. However investors who fail to anticipate health and aged-care costs may be able to avoid ruin and indeed outperform a static investment strategy, if they adopt a form of drawdown dynamic lifecycle (DDLC) investment strategy. This naturally requires a higher risk tolerance than they may be able to bear, but it may also be the only way to avoid ruin.

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